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UNIT V

Digital filters Design Techniques

Design of IIR and FIR digital filters, Impulse invariant and bilinear transformation, windowing techniques rectangular and other windows, examples of FIR filters, design using windowing.

DIGITAL FILTERS DESIGN TECHNIQUES

FIR FILTER

FIR system has finite duration unit sample response. i.e. $h(n) = 0$ for $n < 0$ and $n \geq M$. Thus the unit sample response exists for the duration from 0 to $M-1$. FIR systems are non recursive. Thus output of FIR filter depends upon present and past inputs.

Difference equation of the LSI system for FIR filters becomes

$$y(n) = \sum_{k=0}^M b_k x(n-k)$$

FIR systems has limited or finite memory requirements. FIR filters are always stable FIR filters can have an exactly linear phase response so that no phase distortion is introduced in the signal by the filter. The effect of using finite word length to implement filter, noise and quantization errors are less severe in FIR than in IIR.

FIR filters can have an exactly linear phase response so that no phase distortion is introduced in the signal by the filter.

The effect of using finite word length to implement filter, noise and quantization errors are less severe in FIR than in IIR. FIR filters are generally used if no phase distortion is desired.

IIR FILTER:

IIR system has infinite duration unit sample response. i. e $h(n) = 0$ for $n < 0$ Thus the unit sample response exists for the duration from 0 to ∞ . IIR systems are recursive. Thus they use feedback. Thus output of IIR filter depends upon present and past inputs as well as past outputs

Difference equation of the LSI system for IIR filters becomes

$$y(n) = -\sum_{k=1}^N a_k y(n-k) + \sum_{k=0}^M b_k x(n-k)$$

IIR system requires infinite memory.

IIR filter is usually more efficient design in terms of computation time and memory requirements. IIR systems usually require less processing time and storage as compared with FIR.

IIR filter is usually more efficient design in terms of computation time and memory requirements. IIR systems usually require less processing time and storage as compared with FIR.

Analogue filters can be easily and readily transformed into equivalent IIR digital filter. But same is not possible in FIR because that have no analogue counterpart. IIR filters are generally used if sharp cutoff and high throughput is required.

IIR FILTER DESIGN

- 1. IMPULSE INVARIANCE**
- 2. BILINEAR TRANSFORMATION**
- 3. BUTTERWORTH APPROXIMATION**

IIR FILTER DESIGN - IMPULSE INVARIANCE METHOD

Impulse Invariance Method is simplest method used for designing IIR Filters.

Important Features of this Method are

In impulse variance method, Analog filters are converted into digital filter just by replacing unit sample response of the digital filter by the sampled version of impulse response of analog filter. Sampled signal is obtained by putting $t=nT$ hence

$$h(n) = h_a(nT), \quad n=0,1,2,\dots$$

where $h(n)$ is the unit sample response of digital filter and T is sampling interval.

But the main disadvantage of this method is that it does not correspond to simple algebraic mapping of S plane to the Z plane. Thus the mapping from analog frequency to digital frequency is many to one. The segments $(2k-1)\pi/T \leq \Omega \leq (2k+1)\pi/T$ of $j\Omega$ axis are all mapped on the unit circle $-\pi \leq \omega \leq \pi$. This takes place because of sampling.

Frequency aliasing is second disadvantage in this method. Because of frequency aliasing, the frequency response of the resulting digital filter will not be identical to the original analog frequency response.

Because of these factors, its application is limited to design low frequency filters like LPF or a limited class of band pass filters.

RELATIONSHIP BETWEEN Z PLANE AND S PLANE

Z is represented as $re^{j\omega}$ in polar form and relationship between Z plane and S plane is given as $Z=e^{ST}$ where $s=\sigma + j\Omega$.

$$Z = e^{ST} \quad (\text{Relationship Between Z plane and S plane})$$

$$\begin{aligned} Z &= e^{(\sigma + j\Omega)T} \\ &= e^{\sigma T} \cdot e^{j\Omega T} \end{aligned}$$

Comparing Z value with the polar form we have.

$$r = e^{\sigma T} \quad \text{and} \quad \omega = \Omega T$$

Here we have three conditions

- 1) If $\sigma = 0$ then $r=1$
- 2) If $\sigma < 0$ then $0 < r < 1$
- 3) If $\sigma > 0$ then $r > 1$



STANDARD RELATIONS IN IIR DESIGN

Sr No	Analog System Function	Digital System function
1	$\frac{1}{s - a}$	$\frac{1}{1 - e^{aT} z^{-1}}$
2	$\frac{s + a}{(s+a)^2 + b^2}$	$\frac{1 - e^{-aT} (\cos bT) z^{-1}}{1 - 2e^{-aT} (\cos bT) z^{-1} + e^{-2aT} z^{-2}}$
3	$\frac{b}{(s+a)^2 + b^2}$	$\frac{e^{-aT} (\sin bT) z^{-1}}{1 - 2e^{-aT} (\cos bT) z^{-1} + e^{-2aT} z^{-2}}$

IIR FILTER DESIGN - BILINEAR TRANSFORMATION METHOD:

The method of filter design by impulse invariance suffers from aliasing. Hence in order to overcome this drawback bilinear transformation method is designed. In analogue domain frequency axis is an infinitely long straight line while sampled data z plane it is unit circle radius. The bilinear transformation is the method of squashing the infinite straight analog frequency axis so that it becomes finite.

IMPORTANT FEATURES OF BILINEAR TRANSFORM METHOD ARE:

- Bilinear transformation method (BZT) is a mapping from analog S plane to digital Z plane. This conversion maps analog poles to digital poles and analog zeros to digital zeros. Thus all poles and zeros are mapped.
- This transformation is basically based on a numerical integration techniques used to simulate an integrator of analog filter.
- There is one to one correspondence between continuous time and discrete time frequency points. Entire range in Ω is mapped only once into the range $-\pi \leq \omega \leq \pi$.
- Frequency relationship is non-linear. Frequency warping or frequency compression is due to non-linearity. Frequency warping means amplitude response of digital filter is expanded at the lower frequencies and compressed at the higher frequencies in comparison of the analog filter.
- But the main disadvantage of frequency warping is that it does change the shape of the desired filter frequency response. In particular, it changes the shape of the transition bands.

DIFFERENCE BETWEEN IMPULSE INVARIANCE BILINEAR TRANSFORMATION

IMPULSE INVARIANCE	BILINEAR TRANSFORMATION
In this method IIR filters are designed having a unit sample response $h(n)$ that is sampled version of the impulse response of the analog filter.	This method of IIR filters design is based on the trapezoidal formula for numerical integration.
In this method small value of T is selected to minimize the effect of aliasing.	The bilinear transformation is a conformal mapping that transforms the $j\Omega$ axis into the unit circle in the z plane only once, thus avoiding aliasing of frequency components.
They are generally used for low frequencies	For designing of LPF, HPF and almost all types of Band

like design of IIR LPF and a limited class of band pass filter	pass and band stop filters this method is used.
Frequency relationship is linear.	Frequency relationship is non-linear. Frequency warping or frequency compression is due to non-linearity.
All poles are mapped from the s plane to the z plane by the relationship	All poles and zeros are mapped.

EXAMPLE:

Q.1 Use backward difference for the derivative to convert the analog low pass filter with the system function

$$H(s) = \frac{1}{(s+0.1)^2+9}$$

Sol. $H(z)=H(s)$ at $s=\frac{1-z^{-1}}{T}$

$$H(z) = \frac{1}{\left(\frac{1-z^{-1}}{T}+0.1\right)^2+9}$$

$$T=1S$$

$$H(z) = \frac{1}{(1-z^{-1}+0.1)^2+9} = \frac{1}{(1.1-z^{-1})^2+9} = \frac{1}{1.21+z^{-2}+9-2.2z^{-1}}$$

$$= \frac{1}{z^{-2}-2.2z^{-1}+10.21}$$

Q.2 Using bilinear transformation obtain H(z) if

$$H(s) = \frac{1}{(s+1)^2}$$

And $T = 0.1s$.

SOLUTION:

For Bilinear transformation

$$s = \frac{2(z-1)}{T(z+1)}$$

$H(z) = H(s)$ where

$$H(z) = \frac{1}{\left(\frac{2(z-1)}{T(z+1)} + 1 \right)^2}$$

Substituting $T = 0.1$ s

$$H(z) = \frac{1}{\left(\frac{2(z-1)}{(z+1)} + 1 \right)^2} = \frac{(z+1)^2}{(21z-19)^2}$$

Further simplifying

$$H(z) = \frac{0.0476 (z^{-1}+1)^2}{(1-0.9048z^{-1})^2}$$

BUTTERWORTH FILTERS

The Butterworth low pass filter has magnitude response given by

$$|H(j\Omega)| = \frac{A}{\left[1 + \left(\frac{\Omega}{\Omega_c} \right)^2 \right]^{0.5}}$$



FIR FILTER DESIGN

Features of FIR Filter

FIR filter always provides linear phase response. This specifies that the signals in the pass band will suffer no dispersion. Hence when the user wants no phase distortion, then FIR filters are preferable over IIR. Phase distortion always degrades the system performance. In various applications like speech processing, data transmission over long distance FIR filters are more preferable due to this characteristic.

FIR filters are most stable as compared with IIR filters due to its non feedback nature.

Quantization Noise can be made negligible in FIR filters. Due to this sharp cutoff FIR filters can be easily designed.

Disadvantage of FIR filters is that they need higher order for similar magnitude response of IIR filters.

Difference equation of FIR filter of length M is given as

$$y(n) = \sum_{k=0}^{M-1} b_k x(n-k)$$

And the coefficient b_k are related to unit sample response as

$$H(n) = b_n \text{ for } 0 \leq n \leq M-1$$

$$= 0 \text{ otherwise.}$$

We can expand this equation as

$$Y(n) = b_0 x(n) + b_1 x(n-1) + \dots + b_{M-1} x(n-M+1)$$

System is stable only if system produces bounded output for every bounded input. This is stability definition for any system.

Here $h(n) = \{b_0, b_1, b_2, \dots\}$ of the FIR filter are stable. Thus $y(n)$ is bounded if input $x(n)$ is bounded. This means FIR system produces bounded output for every bounded input. Hence FIR systems are always stable.

FIR Filter Design Methods

The various method used for FIR Filter design are as follows

1. Fourier Series method
2. Windowing Method $W(n)$

Windowing Technique

Various window functions are used for filter design.

Following are the types of window functions:

Rectangular Window:

The weighting function for the rectangular window is given by

$$W_R = \begin{cases} 1 & \text{for } |n| \leq \frac{M-1}{2} \\ 0 & \text{otherwise} \end{cases}$$

Hamming Window:

The causal Hamming window function is expressed by

$$W_H(n) = \begin{cases} 0.54 - 0.46 \cos \frac{2\pi n}{M-1} & \text{for } 0 \leq n \leq M-1 \\ 0 & \text{otherwise} \end{cases}$$

The non-causal Hamming window function is expressed by

$$W_H(n) = \begin{cases} 0.54 + 0.46 \cos \frac{2\pi n}{M-1} & \text{for } 0 \leq n \leq \frac{M-1}{2} \\ 0 & \text{otherwise} \end{cases}$$

Hanning Window:

The window function of a casual Hanning window is given by

$$W_{\text{Hann}}(n) = \begin{cases} 0.5 - 0.5 \cos \frac{2\pi n}{M-1} & \text{for } 0 \leq n \leq M-1 \\ 0 & \text{otherwise} \end{cases}$$

The window function of a non-causal Hanning window is given by

$$W_H(n) = \begin{cases} 0.5 + 0.5 \cos \frac{2\pi n}{M-1} & \text{for } 0 \leq n \leq \frac{M-1}{2} \\ 0 & \text{otherwise} \end{cases}$$

The width of the main lobe is approximately $8\pi/M$ and the peak of the first side lobe is at -32dB.



Blackman Window function:

The window function of a casual blackman window is expressed by

$$W_B(n) = \begin{cases} 0.42 - 0.5 \cos \frac{2\pi n}{M-1} + 0.08 \cos \frac{4\pi n}{M-1} & \text{for } 0 \leq n \leq \frac{M-1}{2} \\ 0 & \text{otherwise} \end{cases}$$

The window function of a casual blackman window is expressed by

$$W_B(n) = \begin{cases} 0.42 + 0.5 \cos \frac{2\pi n}{M-1} + 0.08 \cos \frac{4\pi n}{M-1} & \text{for } n \leq \frac{M-1}{2} \\ 0 & \text{otherwise} \end{cases}$$

Q.1 Using bilinear transformation design a butterworth filter which satisfies the following condition.

$$0.8 \leq |He^{jw}| \leq 1 \quad 0 \leq w \leq 0.2\pi$$

$$|He^{jw}| \leq 0.2 \quad 0.6\pi \leq w \leq \pi$$

$$\text{Sol. } A_p=0.8, \quad \omega_p=0.2\pi, \quad , \quad A_s=0.8, \quad \omega_s=0.6\pi$$

Assume $T=1S$

$$\text{Thus } \Omega_p=2\tan\frac{0.2\pi}{2}=0.65$$

$$\Omega_s=2\tan\frac{0.6\pi}{2}=2.75$$

Order of the filter

$$N=\frac{1}{2} \frac{\log \left[\frac{1}{A_s^2} - 1 \right]}{\log \frac{\Omega_s}{\Omega_p}} \frac{1}{2} \frac{\log \left[\frac{1}{0.8^2} - 1 \right]}{\log \frac{2.75}{0.65}} = 1.3$$

Thus approximately we can take $N=2$

Q.2 Find the order and cut-off frequency of a digital filter with the following specification. Use impulse invariance method.

$$0.89 \leq |He^{jw}| \leq 1 \quad 0 \leq w \leq 0.4\pi$$

$$|He^{jw}| \leq 0.18 \quad 0.6\pi \leq w \leq \pi$$

$$\text{Sol. } A_p=0.89, \quad \omega_p=0.4\pi, \quad , \quad A_s=0.18, \quad \omega_s=0.6\pi$$

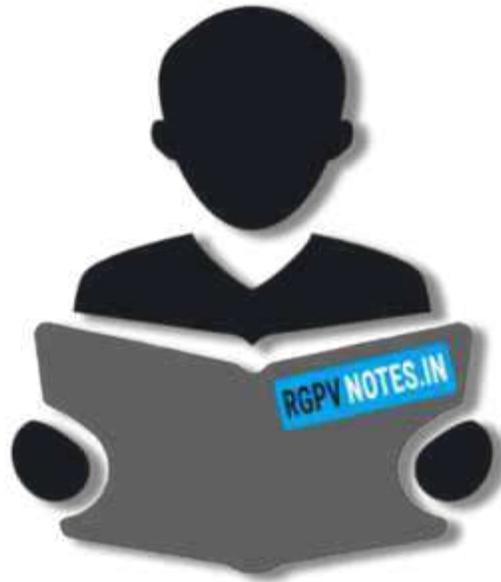
Assume $T=1S$

$$\text{Thus } \Omega_p=\frac{0.4\pi}{1}=0.4\pi$$

$$\Omega_s=\frac{0.6\pi}{1}=0.6$$

Order of the filter

$$N=\frac{1}{2} \frac{\log \left[\frac{1}{A_s^2} - 1 \right]}{\log \frac{\Omega_s}{\Omega_p}} \frac{1}{2} \frac{\log \left[\frac{1}{0.18^2} - 1 \right]}{\log \frac{0.6\pi}{0.4\pi}} = 5.83$$



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